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ABSTRACT

This paper searches for a new growth engine in the new Info-Tech economy. IT-network effects are incorporated into Romer's (1990) framework. Network effects support long-term steady state growth in per capita variables even without innovation, and growth rate increases with network externalities. Networked growth is sub-optimal, so should we break up an IT monopoly? To answer this we compare monopoly, Cournot and Bertrand set-ups. Cournot always ranks last socially, but Bertrand can be superior to monopoly if network effects are strong. When network interacts with Romer's endogenous innovation, growth rate increases, probably by up to a percentage point per year.

Key words: networks, optimal growth, breaking up an IT monopoly

JEL Classification number: O40

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1 INTRODUCTION

It is almost fashionable to talk about *the New Economy* these days. In many ways this simply means the economy of the United State of America. The exceptional performance in computer/peripheral and information technology (IT) sectors ended the 1972-1995 productivity slowdown and produced a record-breaking run of U.S. per capita GDP growth.[1] Jorgenson, Stiroh, Gordon and Sichel (2000) proclaims that "A consensus is now emerging that something fundamental has changed, with 'new economy' proponents pointing to information technology (IT) as the causal factor behind the strong performance." (p.125)

If this new economy is really different from the old then at its core we expect to find a new growth engine driving it. This paper is a theoretical attempt to search for this new engine of growth. In this venture existing growth models have much to offer, having just gone through a decade and a half of revolutionary revision led by Romer (1986, 1990), Lucas (1988), Aghion and Howitt (1992), and many others. Yet, one would be hard pushed to argue that many of these leading models were sufficiently focused on the new economy to address the IT question head on. In some sense they were preoccupied with a deep-seated dissatisfaction with the assumption of exogenous growth, and were happy to find any growth engine for as long as it is *endogenous* and for as long as it supports per capita long-term growth. It seems clear that in order to understand the new economy, some essential features of IT need to be incorporated into its growth mechanism. Which features should be incorporated and how we should do it provide the starting point of our enquiry.

[1] In February 2000, the American economy set a record for the longest business expansion since records began in 1850. For detailed accounts of the ending of the productivity slowdown and the decade-long uninterrupted growth see Jorgenson (2001) and Gordon (2002).

An emerging literature, pertinently portrayed by Shipiro and Varian (1998), argues that a crucial aspect of information technology is its network effect. Such networks are not new, as numerous historical anecdotes would attest.[2] Owning a telephone would be pointless without a network of friends who also own telephones. Networks, however, dramatically increase its prominence in the case of computer/peripherals/software and telecommunication. David (2000) argues that the IT-network revolution is only beginning and we have had merely a taste of what the IT and the Internet may eventually offer.[3]

Networks are a powerful source of externality, and by that a potent source of increasing returns. Consequently, it is a good candidate for providing an engine of growth for the new economy. But increasing returns destroy competition. We saw it doing so with a vengeance in computer and IT industries, giving rise to monopolies like Microsoft, Intel and Yahoo. Romer (1990) deals with non-constant returns by resorting to monopolistic competition in the capital-cum-innovative intermediate goods sector. The same will not suffice for our purpose here. A central conclusion in the network economics literature is that the market structure tends to 'tip' to produce a monopoly. We saw numerous examples such as MS Word defeated WordPerfect word processor, IBM's PC defeated McIntosh personal computers, and Matsushita's VHS totally eliminating Sony's Beta Max in video tape recording. So instead we mimic this by studying growth under a monopoly instead of monopolistically competition supplying the networked intermediate commodity.

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[2] Shapiro and Varian (1999) described achieving a unified network of railway gauges and power standard benefited the U.S. economy in 19th century. More recent examples are the introduction of the telephone the fax machine, and the color television.

[3] David (ibid.) draws a parallel between computer and the dynamo emphasizing their similarity both as general-purpose machines. He points out that dynamo had a diffusion lag took a couple of decades. We should patiently await the full effect of the IT revolution as the use of computers diffuse to the economy at large.

It is easy to imagine that this networked growth equilibrium would fail to produce the socially optimum. It is our purpose in this paper to characterize this optimality, and to examine which parameters and which policies would bring us closer to that ideal. Towards the end of the U.S. Clinton Administration, the anti-trust legislation had taken an aggressive stance against Microsoft. A solution, becoming increasing unlikely under the Bush Administration but remains at least as an option, is to enforce rivalry by breaking up the monopoly. Our model again mimics this, and studies growth when the networked sector becomes a *duopoly*. Our result shows that a Cournot duopoly will always do *worse* than the monopoly, but a Bertrand duopoly may do better if the network effect is sufficiently strong.

The basic model of this paper, presented in Section two below, is closest in spirit to Romer's (1990) endogenous innovation or could instead be called networked growth without innovation. To study social optimality and enforced rivalry (Sections three and four below) we continue to focus, for simplicity, on network effects without innovation. We are able in this endeavor to obtain social second best rankings of monopoly, Cournot and Bertrand duopoly. Furthermore, the interaction of network effects with endogenous innovation is interesting, and we allow network and innovation to interact in Section five. We find, assuming reasonable parameter values, the economy could grow from twenty to even fifty percent faster with network effects, than when there is only endogenous innovation as in Romer (ibid.). From a base of about two percent, network effect could add as much as a full percentage point to long-term per capita growth.

The plan of this paper is as follows. The next section presents the basic model under monopoly. Section three solves the social planner's problem. Section four studies enforced rivalry. Section five examines interactions between networks and innovation. Section six concludes.

2 The Model

The economy consists of a final goods sector employing labor and an intermediate good, which is subject to network effects.[4] The production function for the homogeneous final consumption good Y_i, by firm $i = 1, 2, ...$, at time *t* is

$$Y_{it} = A_t L_{it}^{1-\alpha} K_{it}^{\alpha}. \tag{1}$$

L_{it} is conventional homogeneous labor, of which the country, considered in isolation, is endowed with a stock of L_t at *t*. *K* is 'capital' which is different from traditional machinery in that it is subject to network externality, and it is treated for simplicity as Romer (1990) did as putty-putty.[5] Network externality is embodies in *A*, the productivity factor, expressed as

$$A_t = f(K_t), \quad df(K_t)/dK_t > 0; \quad K_t \equiv \sum_i K_{it}. \tag{2}$$

This says that production technology for final good *Y* increases as more of the same type of capital *K* is in used in aggregate. For sharp results we let

[4] In other words we focus on network effects in the intermediate goods sector and ignore those in the final good sector. This is done primarily for simplicity. To the extent that network effects are also prominent in final consumption goods, our results would be an *underestimate*. On the other hand, as late as 1999 computer and peripherals account for only 3.5 percent of norminal GDP in the nonfarm nonhousing private business economy (Gordon, 2002, p.21). There exists a large portion of intermediate goods that are not subject to strong network effects. By assuming network effect to be strong and uniform throughout our intermediate sector, our result would produce an *overestimate*. With luck these should offset each other at least to some extent.

[5] Romer (1990) justified this assumption by arguing that in the steady state revision of capital investment plans would not be necessary. In the present context we can further justify this by noting that owing to the rapid pace of technological obsolescence and cost reduction, more and more IT-related goods are no longer 'durables'. If obsolescence is sufficiently rapid then investment indeed becomes putty-putty.

$$A_t = K_t^{\beta}, \qquad 0 < \beta < 1.^{6} \tag{2'}$$

The time-subscript will henceforth be suppressed wherever confusion is not likely to arise.

Constant returns to scale ensures perfect competition in *Y*. Each *Y*-producer is too small to influence *K* and thus takes *A* as given. We normalize the price of *Y* to one. Each factor will be paid its marginal products. Denote the prices of *K* and *L* by *p* and *w*, the factor employment decisions are

$$\frac{\partial Y_i}{\partial K_i} = \alpha A L_i^{1-\alpha} K_i^{\alpha-1} = p \tag{3}$$

$$\frac{\partial Y_i}{\partial L_i} = (1-\alpha) A L_i^{-\alpha} K_i^{\alpha} = w. \tag{4}$$

Although individual users of *K* are too small to take equation (2') into account, the seller of *K* is fully aware of such effects.[7] This implies that the production of *K* faces increasing returns and competition will not prevail. As mentioned earlier the information networks literature predicts this *K*-producing market structure to 'tip' in favor of a single winner such as Microsoft, IBM PC and VHS. We shall gloss over this process of 'tipping', and assume a single monopolist producer of *K*, denoted by *'M'*.

Summing over *i* and using (2'), equation (3) gives the derived demand for *K*

$$K = (\alpha/p)^{\frac{1}{1-\alpha-\beta}} L^{\frac{1-\alpha}{1-\alpha-\beta}} \tag{5}$$

[6] We could have modeled network effect as capital-augmenting, e.g. $Y_{it} = L_{it}^{1-\alpha}(AK_{it})^{\alpha}$ where $A = f(\sum K_{it})$. Clearly little will change fundamentally with my simpler specification.

[7] Indeed companies are keen to resort to various strategies to promote and preserve benefits arising from network effects. Airlines' frequent flier program and hotels' frequent guest program are good examples.

where $L \equiv \sum_i L_i$ is the total labor since it is used only in the production of *Y*. Let *L* grow at a constant rate *n*, i.e. $L_t = L_0 e^{nt}$.[8] In order to preserve a downward slopping demand for *K* in (5), network effect needs to be not too large. We shall assume $(1-\alpha) > \beta$ holds.[9]

At each *t* total revenue from the sale of *K* is

$$\alpha K_t^{\alpha+\beta} L^{1-\alpha} = \alpha K_t^{\alpha+\beta} L_t^{1-\alpha} . \tag{6}$$

Suppose one unit of *Y* can be converted into one unit of *K* in each *t*. The monopolist *M*'s decision problem is to find a time-path of capital $\{K_t\}$ to maximize the discounted profit flow

$$\max_{K_\tau} \int_t^\infty [\alpha K_\tau^{\alpha+\beta} L_0^{1-\alpha} e^{(1-\alpha)n\tau} - K_\tau] \cdot e^{-r(\tau-t)} d\tau . \tag{7}$$

We shall take interest rate *r* to be constant, which will in any case be true in the steady state equilibrium (to be verified shortly). Since both revenue and cost are intertemporally separable in (7), *M* treats this as a period-wise static problem by choosing K_t to maximize profits in each *t*, yielding

$$K_t = [\alpha(\alpha+\beta)]^{\frac{1}{1-\alpha-\beta}} L_0^{\frac{1-\alpha}{1-\alpha-\beta}} e^{\left(\frac{1-\alpha}{1-\alpha-\beta}\right)nt} . \tag{8}$$

[8] This is a departure from Romer (1990), who uses innovative investment to drive growth. Lucas (1988) uses human capital investment to drive growth, thus also have little need for population growth. The present framework assumes population growth, on which network hinges and multiplies to generate income growth. This is not degeneration back to the Solow model though, for unlike in the Solow case, network effect is capable of generating income growth *per capita.* We discard population growth in Section five below, where network effect 'hinges' on Romer's endogenous innovation.

[9] The empirical literature generally points to a value of $K_i \to Y_i$ elasticity at individual firm's level of $\alpha \approx 0.3$ (see Walter (1963)). I have not been able to find any empirical measure on β. Condition $(1-\alpha) > \beta$ implies $\beta < 0.7$. This seems perfectly reasonable *a priori*. If a firm hires ten percent more K_i, say, it would only increase firm *i*'s output by only three percent. If instead *other firms* use ten percent more *K*, the external effect would be highly unlikely to be so strong as to increase *i*'s output by seven percent. To estimate the size of β would be an interesting topic for future research.

It follows that $g_K \equiv \dot{K}/K = (1-\alpha)n/(1-\alpha-\beta)$. Per capita capital $k \equiv K/L$ grows at a rate $g_k \equiv \dot{k}/k = \beta n/(1-\alpha-\beta)$. Thus K grows at a constant multiple rate of *n*, and this multiple rises with both α and β. Two points are noted immediately. First, if $\beta = 0$, $\dot{K}/K = n$ and $\dot{k}/k = 0$. Capital just keep pace with population growth but remains constant in per capita terms. Second, $dg_K/d\beta > 0$ and $dg_k/d\beta > 0$. The increasing network externality from the increasing population size prompts the monopoly to sell more *K*, as well as more *K* per capita.

Using (8) in the square-bracketed term in (7) gives *M*'s profit π_t at *t*. Routine calculation shows that $\pi_t = \phi \cdot L_t^{(1-\alpha)/(1-\alpha-\beta)}$ where $\phi = [\alpha(\alpha+\beta)^{(\alpha+\beta)}]^{1/(1-\alpha-\beta)}(1-\alpha-\beta)$. Thus $g_\pi \equiv \dot{\pi}/\pi = (1-\alpha)n/(1-\alpha-\beta) = g_K$. In order to channel profits back into consumption we assume *M* issues shares, which are held evenly by all citizens (L_t), and she distributes all profits via dividends in each period *t*.

Now we are ready to tackle national output and consumption. Though consumers may still desire to even lifetime consumption via interest rate *r*, the consumption each period is fixed by the economy-wide instantaneous budget constraint where *C* and *c* are aggregate and per capita consumption respectively:

$$C \equiv cL = Y - K\ . \tag{9}$$

Using (8) and (2') in (1), national output is

$$Y_t = [\alpha(\alpha+\beta)]^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{1-\alpha}{1-\alpha-\beta}} \tag{10}$$

and it grows at a rate $g_Y = (1-\alpha)n/(1-\alpha-\beta) = g_\pi$.[10] National output per capita, denoted $y_t = [\alpha(\alpha+\beta)]^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{\beta}{1-\alpha-\beta}}$, grows at the rate $g_y = \beta n/(1-\alpha-\beta) = g_k$. Now using (8) and (10) in (9), aggregate consumption is

$$C_t = [1-\alpha(\alpha+\beta)][\alpha(\alpha+\beta)]^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{1-\alpha}{1-\alpha-\beta}} \tag{11}$$

and it grows at

$$g_C \equiv \dot{C}/C = (1-\alpha)n/(1-\alpha-\beta) = g_Y = g_\pi . \tag{12}$$

Per capita consumption is

$$c_t = [1-\alpha(\alpha+\beta)][\alpha(\alpha+\beta)]^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{\beta}{1-\alpha-\beta}} , \tag{13}$$

and it grows at

$$g_c \equiv \dot{c}_t/c_t = \beta n/(1-\alpha-\beta) = g_y = g_k . \tag{14}$$

Thus the income and consumption paths are determined by the network structure and the monopolist's profit-maximizing output choice, without reference to consumers' intertemporal maximization. Interest rate plays a secondary role of reconciling consumers' desire to smooth lifetime consumption, with the predetermined consumption path given in (14). Suppose we adopt the standard Ramsey utility for per capita consumption c_t

$$U = \int_0^\infty \left(\frac{c_t^{1-\theta} - 1}{1-\theta} \right) \cdot e^{-\rho t} dt \tag{15}$$

which yields the familiar household intertemporal optimization condition $\dot{c}/c = (r-\rho)/\theta$. Using (14) and rearranging, *r* is indeed a constant as we claimed earlier

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[10] For orders of magnitude, take $\alpha = .3$, L = one million, if $\beta = .1$ then $Y = 2.14 * 10^6$; if $\beta = .2$ then $Y = 3.2 * 10^7$. This is a fourteen-fold increase in national income when the network elasticity

$$r = \rho + \frac{\beta n \theta}{1-\alpha-\beta}. \tag{16}$$

The economy must maintain the interest rate according to (16). Recall that θ is the constant intertemporal elasticity of substitution in consumption. A higher θ (closer to zero) reflects a stronger desire to smooth consumption overtime. If $r > \rho + \frac{\beta n \theta}{1-\alpha-\beta}$ holds, consumption are growing slower than consumer desires, whose attempt to save and to lend then pushes *r* downwards. Conversely if $r < \rho + \frac{\beta n \theta}{1-\alpha-\beta}$ the attempt to borrow pushes *r* up.

The most striking result of this simple model is two-fold, as it is revealed by equation (14). First, unlike the Solow model, per capita variables exhibit positive long-term steady state growth. Second, this long-term steady state growth rate rises with network externality β. If the new economy, which we argue above is characterized by greater network externality than hitherto, then we should expect per capita growth rates of consumption and national income to experience a permanent rather than a transient increase in its long-term steady state. The speed limit is indeed higher in the new economy, and it rises in tandem with network externality β.

It would again be interesting to obtain some order of magnitude of growth rates we have obtained in theory so far. Suppose we can take $\alpha \approx .3$ and a one percent population growth. Though the restriction $(1-\alpha) > \beta$ allows $0 < \beta < 0.7$, we use as an exercise $\beta \approx 0.3$. Then a network elasticity of 0.3 leads to three-quarter of a percentage point per capita income growth (see Figure 1a). Further, doubling population growth would exactly double $\dot{y}/y$ for every β (see Figure 1b). If network effect β increases

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doubles!

from 0.3 to 0.5, growth rate rises three-fold from 0.0075 to 0.025. Suppose population growth is instead two percent. As β increases from 0.3 to 0.5, growth rate rises three-fold from 0.015 to 0.05.

Put Figures 1a and 1b about here

So our first conclusion is that network effect is indeed capable of generating long-term per capita growth, and it does so by 'hinging' on population growth, which is, by itself, incapable of generating per capita income and consumption growth.

Our conclusion can be put in a different way. Network effect *leverages* on existing dynamic momentums such as population and multiplies it to give rise to long-term growth in per capita variables. This in a sense represents a new category of growth models that we have not encountered in the literature hitherto. The new economy thrives on IT, which provides a wealth of network effects much richer than what we have experienced before. It is more than likely that the multiplier effect just described is useful in explaining the recent growth experience in the United States.

3 Social Optimality

In the model just described, externality causes market failure on two counts. In the first place each competitive *Y*-producing firm fails to recognize the external economies its usage of K_i has on the network community. In the second place, the monopolist producer of *K* supplies too little *K* resulting in a customary 'triangular' deadweight loss. Both failures can be corrected if a social planner supplies *K* instead of the private monopolist.

The initial set up for the social planner is identical to the one in Section two, and equations (1) to (5) continue to hold. Since neither of the market failures is intertemporal in nature, all that the social planner needs to do is to choose K_t to maximize the area between the derived demand curve (5) and the marginal cost (=1) of producing K_t in each period *t*. Setting $p = 1$ in equation (5), the social planner (asterisked) supplies $K_t^* = \alpha^{\frac{1}{1-\alpha-\beta}} L_t^{\frac{1-\alpha}{1-\alpha-\beta}}$, and in per capita terms $k_t^* = \alpha^{\frac{1}{1-\alpha-\beta}} L_t^{\frac{\beta}{1-\alpha-\beta}}$, both are $K^*/K = k^*/k = (\alpha+\beta)^{1/(\alpha+\beta-1)}$ times the monopolist's supply of K_t and k_t, for all *t*.[11] Notice $(\alpha+\beta)^{1/(\alpha+\beta-1)}$ *falls* as β rises (see Figure 2). In fact $\lim_{\beta\to\infty}(\alpha+\beta)^{1/(\alpha+\beta-1)} = 1$.[12] The reason is that as network externality expands, the monopoly's incentive to increase output increasingly offsets its urge to restrict sales. Network effect *to a certain extent* moderates the difference between socially optimal and the private monopoly path (which are parallel, see below) for the supply of *K*.

Put Figure 2 about here

It follows that the private and the social *growth rates* for capital coincide, i.e. $g_K^* = (1-\alpha)n/(1-\alpha-\beta) = g_K$, and $g_k^* = \beta n/(1-\alpha-\beta) = g_k$.

Substituting K_t^* into (1), the socially optimal output path is

[11] If $\alpha = .3$, $\beta = .1$, $K^*/K = 4.6$; for $\alpha = .3$, $\beta = .2$, $K^*/K = 4$. Interestingly, the degree of suboptimality *falls* as network effect is stronger.

[12] $\frac{d(\alpha+\beta)^{1/(\alpha+\beta-1)}}{d\beta} = -\frac{(\alpha+\beta)^{-2+\alpha+\beta/1-\alpha-\beta}}{(1-\alpha-\beta)^2}[1+(\alpha+\beta)\cdot(\log(\alpha+\beta)-1)] < 0$. The last square bracketed term is positive since both α and β are less than one. It is of some interest to note too that $\lim_{\beta\to\infty}(\alpha+\beta)^{1/(\alpha+\beta-1)} = 1$. Had network effect been 'infinitely strong' (think of this in a purely

$$Y_t^* = \alpha^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{1-\alpha}{1-\alpha-\beta}}, \tag{17}$$

and in per capita terms

$$y_t^* = \alpha^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{\beta}{1-\alpha-\beta}}. \tag{18}$$

Compared to the monopoly path, $Y^*/Y = K^*/K = (\alpha+\beta)^{1/(\alpha+\beta-1)}$. The private and social income growth rates again coincide.

Substituting Y_t^* and K_t^* into (9), the socially optimal consumption path is

$$C_t^* = (1-\alpha)\alpha^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{1-\alpha}{1-\alpha-\beta}} \tag{19}$$

and in per capita terms

$$c_t^* = (1-\alpha)\alpha^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{\beta}{1-\alpha-\beta}}. \tag{20}$$

The pattern under social choice compared to the equilibrium under monopoly is now clearly discernable. All the relevant growth rates coincide, in other words the socially optimal paths *parallel* those under monopoly. *The second conclusion in this paper is that networked growth rates of income and consumption are in fact socially optimal, but the monopoly paths are everywhere below the socially optimal paths.* Take consumption for instance. Comparing (19) and (20) with (11) and (13),

$\dfrac{C_t^*}{C_t} = \dfrac{c_t^*}{c_t} = \dfrac{(1-\alpha)}{[1-\alpha(\alpha+\beta)]}(\alpha+\beta)^{\frac{-\alpha-\beta}{1-\alpha-\beta}}$ which measures the degree of consumption sub-optimality arising from the two types of market failure, namely network externalities and monopoly deadweight loss. Again to get a sense of the order of magnitude we taking $\alpha = 0.3$, at $\beta = 0.2$, $c_t^*/c_t \approx 1.64$ (see Figure 3). Consumption can be drastically below

heuristic sense, though it violates the condition $(1-\alpha) > \beta$), then the social planner's and the monopoly's supplies of capital coincide.

the socially optimal rate.[13] It is well justified, therefore, for anti-trust legislation in the United States, and indeed elsewhere, to take a tough stance against monopolies that thrive on network effects. The doctrine of Second Best tells us, of course, that we should be careful as to which specific policy measure would improve rather than worsen welfare. Instead of searching for the optimal policy, we examine in the next section a particularly but relevant policy. The attempt to break up Microsoft amounts to enforcing rivalry into the network industry. The resulting competition will however only be among a few. We examine this below under a duopoly setting.

<u>Put Figure 3 about here</u>

4 Enforced Rivalry

In order to study enforce rivalry, we focus on two alternative duopoly structures, namely Cournot and Bertrand. Our aim is to compare the duopoly equilibrium with monopoly, and with the social optimum.

Denote the two Cournot *K*-suppliers by a subscript $C = 1, 2$. The network equation (2') now takes the form $A = (K_C)^{\beta}$ where K_C is output sold by company *C*. It implies that network effect is completely supplier-specific.[14,15] *Y*-producers continue to take *A* as

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[13] In fact $\lim_{\beta\to 0} \dfrac{C_t^*}{C_t} = \lim_{\beta\to 0} \dfrac{c_t^*}{c_t} = \lim_{\beta\to 0}\left\{\dfrac{(1-\alpha)}{[1-\alpha(\alpha+\beta)]}(\alpha+\beta)^{\frac{-\alpha-\beta}{1-\alpha-\beta}}\right\} = \dfrac{\alpha^{\alpha/\alpha-1}}{1+\alpha} \approx 1.29$.

[14] We need this assumption to keep the picture simple and clear. Technically network effect can either be curtailed or enhanced by making one's product compatible or not compatible with rivals'. Both strategies have been used in practice. For instance, Sun Microsystems was keen to license Java to everyone, even including its arch competitor, Microsoft. But the latter was careful to retain its right to "improve" Java in the licensing agreement, and then went on to add its own "improvements" that only worked on the Windows platform.

given. Summing (3) over each group of K-users, and let L_C denote total labor used in each group ($L_1 + L_2 = L$), the demand for each duopoly company is

$$K_C = (\alpha/p)^{\frac{1}{1-\alpha-\beta}} L_C^{\frac{1-\alpha}{1-\alpha-\beta}}. \tag{21}$$

Taking his rival's output as given, firm C maximizes profits $(pK_C - K_C)$ in each t, implying

$$K_C = [\alpha(\alpha+\beta)]^{\frac{1}{1-\alpha-\beta}} L_C^{\frac{1-\alpha}{1-\alpha-\beta}}. \tag{22}$$

Both Cournot (and later Bertrand) firms produce the same output despite its firm-specific network effect, and both share the technology of converting one Y into one K at each t. Imposing the implied symmetry condition $K_1 = K_2 = K_C$, the Cournot equilibrium output of each firm is

$$K_C = [\alpha(\alpha+\beta)]^{\frac{1}{1-\alpha-\beta}} \left(\frac{L}{2}\right)^{\frac{1-\alpha}{1-\alpha-\beta}}. \tag{23}$$

Total K supplied is $2K_C$. Comparing with (8), the Cournot structure supplies $2^{-\beta/(1-\alpha-\beta)}$ times the monopoly output. For $\alpha = .3$ and $\beta = .2$, $2^{-\beta/(1-\alpha-\beta)} = .75$, implying *less* K is sold than under the monopoly (see Figure 4). This contrasts with the conventional picture where Cournot competition raises total output over the monopolist's. The reason behind this is as follows. Each firm serves only half of the market and enjoys half of the network externality. The reduced network effect more than offsets the competition effect leading to smaller output in the aggregate. Figure 4 shows

[15] It would be interesting to investigate more sophisticated versions of competing networks, for example $A = K_C^{\beta} K_{\mathbb{C}}^{\tilde{\beta}}$, where $K_{\mathbb{C}}$ ($\mathbb{C}$ = 'not C') is the total capital of the rival's type in use, and $0 < \tilde{\beta} < \beta < 1$. To keep our scope manageable we choose $\tilde{\beta} = 0$.

also that relative supply of *K* under Cournot falls when network effect is more pronounced.

Put Figure 4 about here

The Cournot equilibrium income and consumption are

$$Y_C = 2^{\frac{-\beta}{1-\alpha-\beta}}[\alpha(\alpha+\beta)]^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{1-\alpha}{1-\alpha-\beta}};$$

$$C_C = 2^{\frac{-\beta}{1-\alpha-\beta}}[1-\alpha(\alpha+\beta)][\alpha(\alpha+\beta)]^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{1-\alpha}{1-\alpha-\beta}}; \tag{24}$$

and in per capita terms

$$y_C = 2^{\frac{-\beta}{1-\alpha-\beta}}[\alpha(\alpha+\beta)]^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{\beta}{1-\alpha-\beta}};$$

$$c_C = 2^{\frac{-\beta}{1-\alpha-\beta}}[1-\alpha(\alpha+\beta)][\alpha(\alpha+\beta)]^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{\beta}{1-\alpha-\beta}}. \tag{25}$$

Capital and income under Cournot are again a fraction $2^{-\beta/(1-\alpha-\beta)}$ of their counter part under monopoly. The same is true in per capita terms. Their growth rates, however, are identical to those under monopoly. As argued above this is because the competitive incentive to provide more output than the monopoly is more than outweighed by the opposite effect from the lost in network externalities.

This has important Second Best implications on policy, and this constitutes the third important conclusion of this paper. *As an insight from Second Best, enforced rivalry in the form of Cournot would have led us further away from the social optimum*. Further, referring again to Figure 4, the degree of sub-optimality increases with the extent of network externality β.

Now we turn to Bertrand. The two Bertrand K-suppliers are denoted $B = 1, 2$. Network technology continues to be supplier-specific, i.e. $A = (K_B)^{\beta}$. Bertrand competition reduces the price of K to marginal cost, which is one in each t. Using this in (21) and imposing symmetry,

$$K_B = K_1 = K_2 = \alpha^{\frac{1}{1-\alpha-\beta}} \left(\frac{L}{2} \right)^{\frac{1-\alpha}{1-\alpha-\beta}}. \tag{26}$$

National output and consumption are

$$Y_B = 2^{\frac{-\beta}{1-\alpha-\beta}} \alpha^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{1-\alpha}{1-\alpha-\beta}};\ C_B = 2^{\frac{-\beta}{1-\alpha-\beta}} (1-\alpha) \alpha^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{1-\alpha}{1-\alpha-\beta}}, \tag{27}$$

and in per capita terms

$$y_B = 2^{\frac{-\beta}{1-\alpha-\beta}} \alpha^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{\beta}{1-\alpha-\beta}};\ c_B = 2^{\frac{-\beta}{1-\alpha-\beta}} (1-\alpha) \alpha^{\frac{\alpha+\beta}{1-\alpha-\beta}} \cdot L_t^{\frac{\beta}{1-\alpha-\beta}}. \tag{28}$$

Intriguingly, while the Cournot equilibrium levels of income and consumption are a fraction $2^{-\beta/(1-\alpha-\beta)}$ of their monopolist counter parts, from (27) and (28) the Bertrand equilibrium levels of income and consumption are a fraction $2^{-\beta/(1-\alpha-\beta)}$ of their *social optimality counter parts*. Cournot, Bertrand and the monopoly are all sub-optimal, but what are their rankings in terms of (sub-) optimality? It is clear that Bertrand compares favorably with Cournot. The question is whether Bertrand is socially superior to monopoly to justify enforced rivalry? It turns out that the answer depends on the extent of network externalities.

Using (27) and (10), $\frac{Y_B}{Y} = \frac{y_B}{y} = 2^{\frac{-\beta}{1-\alpha-\beta}} (\alpha+\beta)^{\frac{-\alpha-\beta}{1-\alpha-\beta}}$. Take $\alpha = .3$, $\frac{Y_B}{Y} = 1$ if $\beta \approx .38$ (see Figure 5). As Figure 5 shows, aggregate and per capita income are higher along the monopoly path than Bertrand's if network effect is more pronounced, and conversely.

Put Figures 5 and 6 about here

Similar conclusion holds for consumption. Using (28), (11) and (13),

$\frac{C_B}{C} = \frac{c_B}{c} = 2^{\frac{-\beta}{1-\alpha-\beta}} \frac{(1-\alpha)}{[1-\alpha(\alpha+\beta)]} (\alpha+\beta)^{\frac{-\alpha-\beta}{1-\alpha-\beta}}$. Take $\alpha = .3$, $\frac{C_B}{C} = 1$ if $\beta \approx .338$ (see Figure 6). It shows also that aggregate and per capita consumption are higher along the monopoly path than Bertrand's if network effect is more pronounced, and conversely.

Our fourth conclusion in this paper may be put as follows. *By enforcing rivalry, the ranks of alternative market set-ups are, firstly, Cournot is invariably the worst amongst the three; secondly, Bertrand* $\succ$ *monopoly if network effects are sufficiently strong, but Bertrand* $\prec$ *monopoly otherwise.* In general, anti-trust policies need to be carefully devised and sweeping judgments are not supported by theory. In the event of breaking up a monopoly such as Microsoft, for instance, is more likely to improve welfare if the resulting price competition is fierce, and if network effects are sufficiently overwhelming.

5 Network-cum-Innovative Growth

We now return to a task set aside earlier, namely of building networked growth not on population growth, but on Romer's (1990) endogenous innovation. This exercise is important not only comparison reasons, but also it allows us to study the interactions between networks and innovation. For simplicity we follow Barro and Sala-I-Martin's (1995) interpretation of Romer (ibid.). We shall modify Romer's and our model where appropriate to preserve steady state growth.

Suppose network effect is associated with the number of intermediate products available.[16] The production function for firm i is

$$Y_i = N^{\beta} \cdot L_i^{1-\alpha} \sum_{j=1}^{N} (X_{ij})^{\alpha}, \qquad \beta > 0. \tag{29}$$

The demand equations for intermediate goods and labor are

$$X_{ij} = N^{\frac{\beta}{1-\alpha}} L_i (\alpha / P_j)^{\frac{1}{1-\alpha}}; \tag{30}$$

$$L_i = (1-\alpha) \cdot (Y_i / w). \tag{31}$$

A representative intermediate producer j, being a monopoly protected by an infinite patent, converts a unit of final good Y into a unit of X, and earns present value returns

$$V(t) = \int_t^{\infty} (P_j - 1) \cdot X_j \cdot e^{-r\cdot(v-t)} dv \tag{32}$$

where interest rate r is constant in equilibrium.

We abandon labor growth in our model and adopt Romer's constant population L. Summing (30) over users i, the aggregate demand for intermediate j is,

$$X_j = N^{\frac{\beta}{1-\alpha}} L (\alpha / P_j)^{\frac{1}{1-\alpha}}. \tag{33}$$

Firm j chooses P_j to maximize $(P_j - 1) \cdot X_j$ at each date in $V(t)$. The choice is

$$P_j = 1/\alpha. \tag{34}$$

Feeding this back into (33),

$$X_j = N^{\frac{\beta}{1-\alpha}} L \alpha^{\frac{2}{1-\alpha}}. \tag{35}$$

Assume monopoly j, in evaluating (34), takes only the existing network effect (N_t) into account, and does not form any expectation on the change of N_t. More

[16] We may think of them as complementary goods, either of the hardware-hardware or hardware-software

importantly, each *j* is too small to recognize it's own impact on N_t when deciding to enter the industry. This pair of assumptions seems not too strong, reminiscing simply the nature of monopolistic competition. Using (34) and (35), we get

$$V(t) = L \cdot N^{\frac{\beta}{1-\alpha}} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{\frac{2}{1-\alpha}} \cdot \int_t^{\infty} e^{-r\cdot(v-t)} dv . \tag{36}$$

Now we come to the crux of network-cum-innovative growth. If we follow Romer (ibid.) and assume a constant cost (η) of innovative entry into the intermediate good industry, long-term steady state will not be possible. The growth in *N* enhances network effect, which in turn increases the returns to innovative activity, inducing a forever increasing fraction of resources to be poured into innovative activity. To see this more clearly we press on for a moment with a constant η. Equating it with $V(t)$ we have

$$r = \left(\frac{L}{\eta}\right) \cdot N^{\frac{\beta}{1-\alpha}} \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{\frac{2}{1-\alpha}} . \tag{37}$$

Interest rate rises with the number of intermediates *N*, instead of being a constant as is expected in a steady state. If we follow through with constant η it is easy to show that consumption and income growth would not be constant either.

In order to counter the accelerating incentive to invest, the obvious modification is to make innovative cost η rise with innovation; i.e. $\eta = f(N)$ and $f' > 0$.[17] More specifically, in order to keep interest rate *r* constant, η must rise at exactly the rate $\frac{\beta}{1-\alpha} \cdot \gamma_N$. We assume therefore

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variety.

[17] There is no a priori reason to believe that the innovative cost is linear (e.g. Scherer (1980), and Kamien and Schwartz (1975, 1982)). In fact as a country becomes richer, the cost of inventing a new commodity also rises and in 2002 it most probably costs more, in absolute dollar terms, than in 1902. This is enough to justify our assumption of rising innovative cost.

$$\eta = \sigma \cdot N^{\frac{\beta}{1-\alpha}}, \qquad \sigma > 0. \tag{38}$$

Using this in (37) we have

$$r = \left(\frac{L}{\sigma}\right) \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{\frac{2}{1-\alpha}}. \tag{39}$$

This interest rate and innovative cost (38) will peg the rate of return $V(t)$ to maintain a constant (and assumed positive) rate of innovation γ_N.[18]

Assumption (38) is also sufficient for steady state growth in consumption and income. To see this let households have Ramsey utility $U = \int_0^\infty \left(\frac{c^{1-\theta}-1}{1-\theta}\right) \cdot e^{-\rho t} dt$, the maximization of it yields the growth rate of per capita consumption γ_c and by using (39) it becomes

$$\gamma_c = (1/\theta) \cdot \left[(L/\sigma) \cdot \left(\frac{1-\alpha}{\alpha}\right) \cdot \alpha^{\frac{2}{1-\alpha}} - \rho \right]. \tag{40}$$

Summing (29) across *i*, aggregate output is $Y = N^{1+\beta} \cdot L^{1-\alpha} X^{\alpha}$, and using (35), we have

$$Y = N^{\frac{1-\alpha+\beta}{1-\alpha}} \cdot L\alpha^{\frac{2\alpha}{1-\alpha}} \tag{41}$$

which implies $\gamma_Y = \gamma_y = \dfrac{1-\alpha+\beta}{1-\alpha} \cdot \gamma_N$. Income must grow faster than innovation since the latter is becoming increasingly expensive over time.

The economy's budget constraint is $C = Y - \sigma \cdot N^{\frac{\beta}{1-\alpha}} \cdot \gamma_N \cdot N - NX$. Using (41) and (35), we get

[18] In principle we can build a model of investment and borrowing to further justify our assumption of interest rate determination. We refrain from doing so here to keep our scope manageable.

$$C = N^{\frac{1-\alpha+\beta}{1-\alpha}}\left(L\alpha^{2\alpha/1-\alpha} - \sigma\gamma_N - L\alpha^{2/1-\alpha}\right). \qquad (42)$$

Thus consumption grows at the same rate as income, and everything grows at $\frac{1-\alpha+\beta}{1-\alpha}$ times the rate of innovation. Romer's (ibid) model can be seen as a special case of ours with $\beta \to 0$, when consumption and income both grow at the same rate as innovation.

We state the final result of this paper in some order of magnitude. If $\alpha = .3$, $\beta = \{0.1, 0.2, 0.3\}$ implies networked growth in consumption and income to be $\frac{1-\alpha+\beta}{1-\alpha} = \{1.14, 1.28, 1.42\}$ times *larger* than simple innovative growth a la Romer (see Figure 7). If the economy with Romer's endogenous innovation grows at two percent, say, with network effects it could become $(2.28, 2.56, 2.84\}$ percents depending on the size of β.

Put Figure 7 about here

6 Summary and Conclusions

Our search for a new growth engine for the IT economy has proved fruitful. We are now reasonably sure that network effect can generate growth. It is, moreover, growth engine of a kind that we have not encountered before in the literature. In Section two we saw it 'hinges' on population growth and multiplies it to produce per capita income and consumption growth. In Section five it interacts with endogenous innovation and growth rate rises with network effects. We also studied social optimality and Second Best policy issues. It is perhaps worth putting our five major conclusions together as we end.

First, network effect capable of generating long-term *per capita* growth, and it does so by 'hinging' on population growth, which is, in itself, incapable of generating per capita income and consumption growth (Section two).

Second, networked *growth rates* of income and consumption are in fact socially optimal, but the monopoly paths are everywhere below the socially optimal paths. The degree of sub-optimality can be substantial (Section three).

Third, as an insight from Second Best, enforced rivalry in the form of Cournot would do worse than either monopoly or Bertrand duopoly, thus leading us further away from the social optimum (Section four).

Fourth, again in terms of social welfare, Bertrand is superior to monopoly if network effects are sufficiently strong, but inferior otherwise (Section four).

Fifth, network effect interacts with endogenous innovation increases growth rate. If growth were otherwise two percent, network effect of the 'reasonable magnitude' would add perhaps another percentage point to growth at the most.

So it is justified to say that at least IT network effects have made the new economy fundamentally different from the old by increasing the long-term equilibrium rate of growth. This new growth rate, while being higher than before if network effects were absent or insignificant, is sub-optimal (Figure 3 shows that consumption under monopoly can be almost fifty percent below optimal). Social loss arises not only from network externality, but also from the conventional monopoly deadweight loss.

So it is also justified to ask whether we should take drastic anti-trust measures to alleviate the high degree of social loss. Our conclusion is certainly against breaking up a monopoly if the resulting rivalry is Cournot. We are for it if the enforced rivalry is Bertrand, and if network effect is substantial.

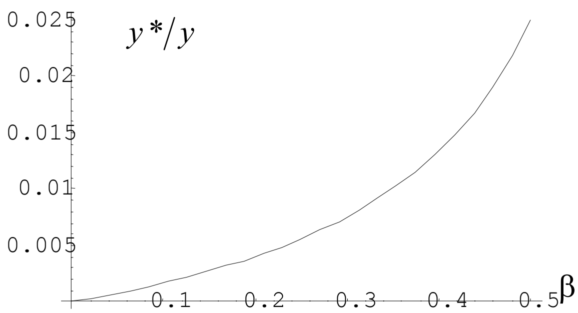

Figure 1a. Per capita income growth rate with $\alpha = .3$, $n = .01$.

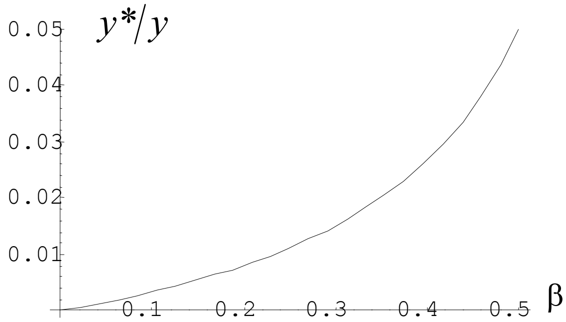

Figure 1b. Per capita income growth rate; $\alpha = .3$, $n = .02$.

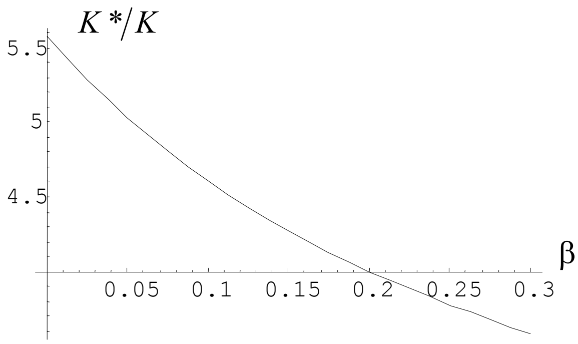

Figure 2. Comparing the socially optimal supply of capital and monopoly's supply, $\alpha = .3$.

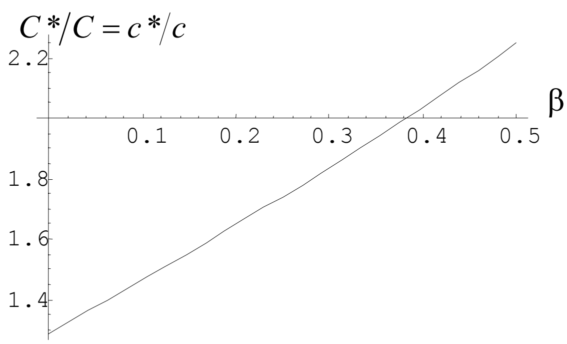

Figure 3. Comparing the socially optimal and monopoly's consumption paths, $\alpha = .3$.

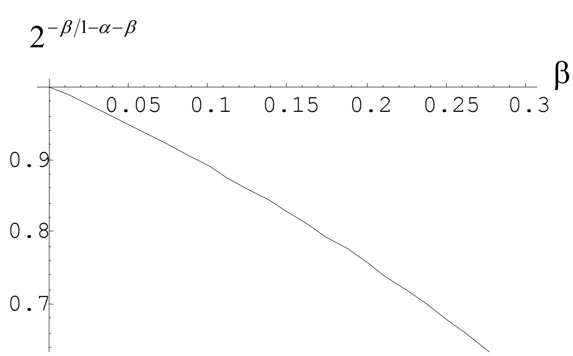

Figure 4. Cournot supplies $2^{-\beta/1-\alpha-\beta}$ times the monopoly supply of K; this figure shows the graph with $\alpha = .3$.

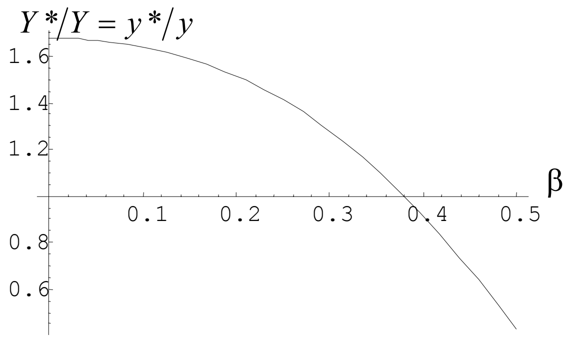

Figure 5. Comparing income paths between Bertrand and monopoly, $\alpha = .3$.

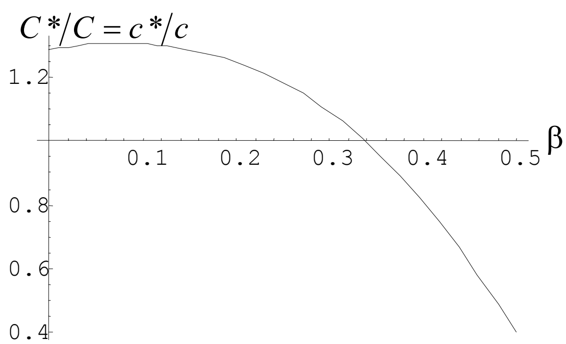

Figure 6. Comparing consumption paths between Bertrand and monopoly, $\alpha = .3$.

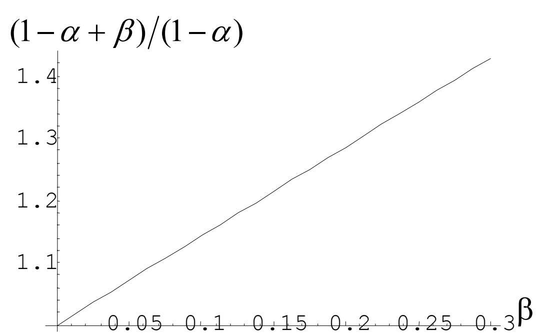

Figure 7. All variables grow at $(1-\alpha+\beta)/(1-\alpha)$ times the rate of (Romer's) innovative growth when network effect is added; the figure shows the graph with $\alpha = .3$.

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